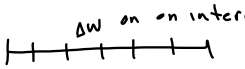
7.5a Applications from Science and Statistics

Work done by a variable force: $W = \int_a^b F(x)\,dx$

F constant

$W = F \cdot d$



Δx - assume F constant on that interval

$\Delta W = F \cdot \Delta x$

$W = \int_a^b F\,dx$

↑ function of x

Dec 17-7:20 PM

A leaky bucket weighs 22N empty. It is lifted from the ground at a constant rate at a point 20m above the ground by a rope weighing 0.4 N/m. The bucket starts with 70N of water but it leaks at a constant rate and just finishes draining as the bucket reaches the top. Find the amount of work done.

empty bucket: $F = 22\ N$ $W = 22 \cdot 20 = 440\ J$

water - Force varies; but must be linear

bottom: $(0, 70)$ top $(20, 0)$

pt slope: $m = \frac{0-70}{20-0} = -\frac{7}{2}$ $y = -\frac{7}{2}(x-0)+70$

$W = \int_0^{20} -\frac{7}{2}x + 70\,dx = 700\ J$

rope: also linear

bottom: $20\ m \cdot .4\frac{N}{m} = 8\ N$ $(0, 8)$

top $(20, 0)$

$F = -\frac{2}{5}(x-0)+8$

$W = \int_0^{20} -\frac{2}{5}x + 8\,dx = 80\ J$

Total work = $80 + 700 + 440$

$= 1220\ J$

Dec 17-7:25 PM

How much work does it take to pump all the water over the rim of a cylindrical tank of height 10ft and diameter 10ft?

water density

$\delta = 62.4\ \frac{lb}{ft^3}$

Δw = work for 1 slice

Δw = Force · distance

= weight · $(10-y)$

= $\delta \cdot \Delta V\ (10-y)$

↑ volume of slice

$= \delta\ \pi \cdot 5^2 \Delta y\ (10-y)$

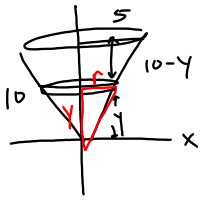
$\Delta W = 62.4 \cdot \pi \cdot 25\ (10-y)\Delta y$

$W = \int_0^{10} 62.4 \cdot \pi \cdot 25 \cdot (10-y)\,dy$

$= 245{,}044\ ft \cdot lbs$

Dec 17-7:29 PM

A conical tank of height and diameter 10ft is filled to within 2 ft of the top with olive oil weighing 57 lb/ft^3. How much work does it take to pump the oil to the rim of the tank?



Δw = weight · distance

= $\delta \cdot \Delta V\ (10-y)$

= $\delta\ (10-y)\ \pi r^2 \Delta y$

$\Delta W = 57\ (10-y)\pi \left(\frac{y}{2}\right)^2 \Delta y$

r is not constant

need r as a function of y

$\frac{r}{y} = \frac{5}{10}$ $r = \frac{y}{2}$

$W = \int_0^8 57\ (10-y)\pi \frac{y^2}{4}\,dy$

$W = 7728\pi$

$= 30{,}561\ ft\text{-}lb$

Dec 17-7:31 PM

Dec 20-8:24 AM